# MATH 590: LINEAR ALGEBRA
## SPRING 2017

HOMEWORK SET # 1: REVIEW OF MATH 290

(1) (a) Find the area of the triangle with vertices $(1,1)$, $(-1,1)$, $(0,-2)$.

(b) Find an equation of the plane passing through the points $(1,-2,1)$, $(-1,-1,7)$, $(2,-1,3)$.

(2) Solve the linear system by Gaussian Elimination with back-substitution or Gauss-Jordan elimination.

$$\begin{aligned} 2x + 2y + 4z &= 6 \\ w \qquad - y - 3z &= -1 \\ 2w + 3x + y + z &= 8 \\ -2w + x + 3y - 2z &= -5. \end{aligned}$$

(3) Consider the matrix

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

Find bases for its

(a) Row space.

(b) Column space.

(c) Null space.

(d) Find the rank and nullity of $A$.

(4) Consider the matrix

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & -3 \end{bmatrix}$$

Find $(A^T + 2I_4)^{-1}$.

(5) Consider the matrix

$$A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(a) Find the eigenvalues of $A$.

(b) Find bases for the eigenspaces of $A$.

(c) Is $A$ diagonalizable? If so, find a matrix $P$ such that $P^{-1}AP$ is a diagonal matrix.

(6) Let $B = \{(1,0,2),\ (0,1,3),\ (1,1,1)\}$ and $B' = \{(2,1,1),\ (1,0,0),\ (0,2,1)\}$.

(a) Find the transition matrix from $B'$ to $B$.

(b) If $[\vec{x}]_{B'} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, find $\vec{x}$.

(c) Use the matrix you obtained in (a) to find $[\vec{x}]_B$

(7) Determine whether the set $S$ is linearly independent or linearly dependent.

(a) $S = \{(1,3,1),\ (0,1,2),\ (1,0,-5)\}$.

(b) $S = \{(4,\ 2,\ 0,\ -2),\ (-2,\ -1,\ 0,\ 1)\}$.

(8) Let $A\vec{x} = \vec{b}$ be the following system.

$$\begin{aligned} 2x + y \quad\quad &= 1 \\ y - z &= 2 \\ -2x + y + z &= -2. \end{aligned}$$

Solve the system by.

(a) Find an $LU$-factorization of $A$.

(b) Solving the lower triangular system $L\vec{y} = \vec{b}$.

(b) Solving the upper triangular system $U\vec{x} = \vec{y}$.