MATH 590: LINEAR ALGEBRA
SPRING 2017

HOMEWORK SET # 2: PROOFS BY INDUCTION AND PROPERTIES OF FIELDS

Recall that $\sum_{i=m}^{n} a_i$ represents the sum $a_m + a_{m+1} + \cdots + a_n$.

(1) Let n be a positive integer. Prove by induction that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

That is $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

(2) Let n be a positive integer. Find a formula for $\sum_{i=1}^{n} \frac{1}{i(i+1)}$ and prove by induction that this formula always holds.

Hint: Compute particular cases, for example $n = 1, 2, 3, 4$, to guess what this formula should be.

(3) Let n be a positive integer such that $n \geqslant 4$. Prove by induction that $n! > 2^n$.

Recall that $n! = 1 \cdot 2 \cdots 3 \cdots n$, for example, $3! = 1 \cdot 2 \cdot 3 = 6$

Hint: Since this statement holds for numbers bigger than or equal to 4, the base case of this induction is $n = 4$.

(4) Let S be a set that has exactly n elements. Prove that the number of subsets of S is 2^n.

Hint: Use induction. The base case is when S has one element, then S has two subsets: the empty set and itself.

(5) Prove that Z_2 is a field (Example 4, page 553), by verifying the given operations satisfy conditions (F 1) – (F 5).

(6) Prove part (a) of Theorem C.1. (Appendix C).

(7) Complete the proof of the Corollary of Theorem C.1.

(8) Prove the Corollary of Theorem C.2.