Homework questions: The Malthusian Model with Physical Capital and Productivity Growth

1. Assume this production function for aggregate output: , where each exponent is positive but less than zero. Hence, the exponents sum to one which means this function has CRS (constant returns to scale) in all the inputs. Technology (A) enters the function in a different way than we’ve done in class so far, and it is called “labor augmenting technical change.” There are reasons to like this form that I may have time to discuss later in the course. Show this production function can be re-written as:.
2. From this last equation and the mathematical rules for growth rates we discussed in class, derive the equation that describes the growth rate of y in terms of parameters and variables that are growing in the model. (Assume: Land is constant and the parameters in the exponents are constant, but all other variables may grow).
3. Suppose the ratio of output to capital, both in terms of per-capita quantities,  is constant. How is the growth rate of y related to the growth rate of k?
4. Write the equation for output growth obtained by combining results from parts C and B.
5. Assume that population growth rate is given by the following function:, where β0 and β1 are positive parameters. This equation simply adds a constant (β0) to the equation we used in class, and is motivated by the same assumption: higher income per-capita induces a family to have more children. Prove that if productivity growth and the parameters are all positive and constant, output eventually reaches a steady state level. What is the steady state value for y in this model in terms of the parameters and the growth rate of productivity?
6. What is in terms of the parameters and the growth rate of productivity?
7. If productivity growth rises to a higher growth rate, what will that do to y and to population growth? Using the graphical model, show and explain what happens in the short run and as the economy evolves to the long run.
8. The parameter is equal the share of income that goes to Land (like α is the share of income for capital). If falls to a lower value, what will happen to y and to population growth? Explain what happens in the short run and as economy evolves to the long run.
9. Western Europe saw its income growth go from about zero before the Industrial Revolution to a small positive number. It also saw population growth go from a small positive rate to a larger rate. Based on results in part G, is it possible for a gradual rise in the productivity growth rate to explain this evidence for Western Europe? Please explain your answer, don’t just say yes or no.
10. Based on results in part H, can a gradual reduction in Land’s share of income explain this evidence for Western Europe? Please explain your answer, don’t just say yes or no.
11. If β1=0 you can’t use the previous solutions to obtain steady state values for y and population growth. Why not? Under this new assumption, what are these steady state values? Hint: everything up to part D is still legitimate, but you have to re-do part E under this new assumption.