Problem Set #1: Univariate Time Series Analysis
Econ 911: Applied Macroeconomics

Unless stated otherwise, $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$ in all of the following questions (a type of white noise).

1. For each of the following 5 univariate stochastic processes, state conditions on model parameters that are required for y to be (i) stationary and (ii) invertible, or state that a process always does or does not satisfy one (or both) of the conditions:

A. $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$

B. $y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

C. $y_t = \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$

D. $y_t = y_{t-1} + \varepsilon_t$

E. $y_t = \varepsilon_t - \varepsilon_{t-1}$

A1. For process A, calculate the population's first partial autocorrelation based on autocovariances for y, where γ_j is the j-th autocovariance (but don't calculate the precise relationships between autocovariances and process parameters).
A2. What are all remaining partial autocorrelations for process A in population?
B1. Calculate all autocorrelations for process B in terms of its parameters in population.
C1. Precisely derive the infinite order moving average representation for process C.
D1. Show that the variance of y_t in process D increases with t.
E1. For process E, what is the first partial autocorrelation in population in terms of process parameters?

2. Suppose x_t is represented by the following process:

$$x_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

Assuming the second order lag polynomial for this moving average process can be factored into two first order lag polynomials with real-valued parameters, find a new stochastic process with different coefficients and a different error variance with precisely the same autocovariances.

[Hint: Section 3.7 in Hamilton's *Time Series Analysis* textbook discusses invertibility and how autocovariance generating functions are helpful in answering this sort of question.]

3. Assume that the true process is:

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

However, instead of estimating an MA model, you estimate an AR(1) model.

a) What is the population OLS coefficient for an AR(1) model in terms of parameters for the true process?
b) What is the ratio of the variance of the OLS residual to the variance of ε for the estimate in part a.
c) Use your equation from part b to make a table that shows this variance ratio for the following values of θ: {0.1, 0.3, 0.5, 0.7, 0.9, 1.0}. What did you learn from this table?

4. Let x_t be generated by:

$$x_t = y_t + e_t \qquad \text{with } e_t \text{ a W.N. process with variance equal to } \sigma_e^2,$$
$$y_t = \rho y_{t-1} + a_t \qquad \text{with } a_t \text{ a W.N. process with variance equal to } \sigma_a^2,$$

with a and e independent from one another and each has a mean of zero.

Derive precisely the univariate ARMA representation for x_t? What type of ARMA process is it?

5. Suppose that y is an AR(p) stochastic process:

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - .... - \phi_p y_{t-p} = \varepsilon_t .$$

Show that if y has a unit root (a 1-L factor) in its autoregressive lag polynomial, then

$$\sum_{i=1}^{p} \phi_i = 1 .$$

[Hints: - A p-th order lag polynomial can be factored into p lag polynomials of order 1.
- Setting L=1 in a lag polynomial provides a useful way of thinking about the sum of coefficients]