

Econ 911: Applied Macroeconomics

Problem Set #1: Univariate Time Series Analysis

Unless stated otherwise, assume $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$ in all of the following questions.

1. For each of the following 5 univariate stochastic processes, answer (i) through (v). State conditions on model parameters that are required to hold if y is (i) stationary and (ii) invertible, or state if the process can not satisfy a condition. Calculate (iii) the autocovariance function; (iv) the autocorrelation function; and (v) the first 3 partial autocorrelations of the process. For parts (iii), (iv) and (v) assume the y process satisfies stationarity and invertibility conditions, unless by construction it is unable to do so. (Note: For B and C below, part v is algebraically tedious)

A. $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$

B. $y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

C. $y_t = \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$

D. $y_t = y_{t-1} + \varepsilon_t$

E. $y_t = \varepsilon_t - \varepsilon_{t-1}$

2. Suppose that x is represented by the following process:

$$x_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

Find a new stochastic process with different coefficients and a different error variance that has precisely the same autocovariances.

3. Show that the follow two conditions:

i) $-.5 \leq \rho_1 \leq .5$

ii) $\rho_i = 0$ for $i = 2, 3, \dots$

are necessary and sufficient for a covariance stationary process x_t to have an MA(1) representation (assuming x is mean zero for convenience).

4. Assume that the true model is MA(1):

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

However, instead of estimating an MA process, you estimate an AR(1) model.

- What is the “population” OLS coefficient for an AR(1) model.
- What is the ratio of the variance of the OLS residual to the variance of ε for the estimate in part a.
- Use your equation from part b to make a table that shows this variance ratio for the following values of θ : {0.1, 0.3, 0.5, 0.7, 0.9, 1.0}. What did you learn from this table?

5. Let x_t be generated by:

$$\begin{aligned}x_t &= y_t + e_t && \text{with } e_t \text{ a W.N. process with variance equal to } \sigma_e^2, \\y_t &= \rho y_{t-1} + a_t && \text{with } a_t \text{ a W.N. process with variance equal to } \sigma_a^2,\end{aligned}$$

where a and e are independent and each has a mean of zero.

- What is the univariate ARMA representation for x_t ?
- Show the relationships between the “population” estimate of the ARMA parameters and the structural parameters of the data generation process.
- How would you test the hypothesis that the variance of e equals zero?

6. Suppose that y is an AR(p) stochastic process:

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \dots - \phi_p y_{t-p} = \varepsilon_t.$$

Show that if y has a unit root (a 1-L factor) in its autoregressive lag polynomial, then

$$\sum_{i=1}^p \phi_i = 1.$$