- ***Question: Solve for the steady state values of capital and output per effective labor unit in our model with population growth and productivity growth, assuming a steady state occurs. (This means solve for the values of endogenous variables in terms of parameters and exogenous variables)***

- Note that: $\dot{\tilde{k}} = \gamma \tilde{k}^{\alpha} - (g + n + \delta)\tilde{k}$

- So set $\dot{\tilde{k}} = 0$ above, and solve for:

$$\tilde{k} = \left[\frac{\gamma}{g + n + \delta} \right]^{\frac{1}{1-\alpha}}$$

- ***Question: Calculate the steady state ratio of K to Y (called the aggregate capital to output ratio). Is it the same as the ratio of k to y or is it different?***

- First I will show that: k/y = K/Y:

$$\frac{K}{Y} = \frac{K/L}{Y/L} = \frac{k}{y}$$

- What is K/Y? in class we showed that:

$$\frac{Y}{K} = \frac{\frac{Y}{AL}}{\frac{K}{AL}} = \frac{\tilde{y}}{\tilde{k}} = \tilde{k}^{\alpha-1}$$

- Therefore $$\frac{K}{Y} = \tilde{k}^{1-\alpha}$$

- Insert in to this the expression for $\tilde{k}$ that we just obtained, to yield:

$$\frac{K}{Y} = \left[\frac{\gamma}{g + n + \delta}\right]$$

- Note that for nearly all values of the parameters that we see for the countries, K/Y is greater than 1. Thus, K>Y, and similarly, k>y.

- ***Question: On a ratio scale, graph steady state values of K and Y (hint the ratio scale and the logarithm scale are the same). How fast are these variables each growing?***
- ***On the same ratio scale, graph steady state values of k and y. How fast are each of these variables growing?***

- First we will solve for growth rates. Then we will plot a graph

- Recall that : $$\tilde{k} = \frac{K}{AL}$$
- If $\tilde{k}$ settles down to a steady state it is not growing. Then from notes in class we know we can take time derivative of the log of the expression to show that:

$$\left(\frac{1}{\tilde{k}}\right)\frac{d\tilde{k}}{dt} = \left(\frac{1}{K}\right)\frac{dK}{dt} - \left(\frac{1}{L}\right)\frac{dL}{dt} - \left(\frac{1}{A}\right)\frac{dA}{dt}$$

- Thus in steady state: $0 = g_K - g_L - g_A$
- Or, $g_K = g_L + g_A$ (note that $g_L = n$ and $g_A = g$, using notation that is common to the literature)
- Using $\tilde{y}$, the same result is shown for Y namely that
- $g_Y = g_L + g_A$

- Note that given $\tilde{k} = \frac{K}{AL}$ and multiplying by A,

  it is easy to see that: $\tilde{k}A = \frac{K}{L} = k$

- Rewrite the equation as $k = \tilde{k}A$
- We know from taking logs and time derivatives that this equation yields that:
- $g_k = g_A$ in the steady state, since $\tilde{k}$ is constant in the steady state.
- Starting with $\tilde{y}$, and using the same

  procedure, it is easy to show: $g_y = g_A$
- Thus if population growth and productivity growth are positive, $g_Y > g_y$

- Using the results for growth rates and for levels of the 4 variables we can graph the steady state relationships over time.

Steady State Values over time: for K, Y, k and y on the ratio scale