Old Test Questions

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1. An important equation from the Solow growth model is,
$\dot{k}_t = sf(k_t) - (g + n + \delta)k_t$. Assume the production function is Cobb-Douglas,
$y_t = f(k_t) = k_t^{\alpha}$.

A. Derive the steady-state value of k in terms of the exogenous parameters: s, n, g, α, δ.

B. Using the Solow model diagram, illustrate and explain the dynamic response of k to δ.

C. Derive the elasticity of steady-state y with respect to δ in terms of parameters

D. Derive the steady-state ratio of capital to output (K/Y).

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1. The Solow model with a Cobb Douglas production function and α=1/3 does a poor job of explaining some key empirical facts compared to when α=2/3 in this model.

A. What are these key empirical facts?

B. Under what conditions would α=share of income going to capital?

C. Describe two explanations that have been given for why α in the aggregate production function could be greater than the share of income going to capital.

2. Consider an economy where consumers live two periods, get utility from consumption and leisure, choose how much labor to supply in the first period and don't work at all in the second period. Consumers choose consumption (c_{1t}), savings (s_t) and labor effort (ℓ_t) when young and consumption when old ($c_{2,t+1}$) to maximize utility which is given by

$$\ln(c_{1t}) + b[\ln(T - \ell_t)] + \frac{\ln(c_{2,t+1})}{1+\rho}$$

with b>0 and ρ>0. We assume that people have T units of time to allocate between labor and leisure. Therefore leisure is written as T-ℓ. Consumers are subject to two budget constraints:

$$s_t + c_{1t} = w_t \ell_t \quad \text{and} \quad c_{2,t+1} = s_t(1 + r_{t+1}) .$$

And real wages (w) and interest rates (r) are taken as given by the consumer.

A. Set up the Lagrangian for the consumer to maximize utility subject to these resource constraints.

B. Obtain first order conditions for this optimization problem (for simplicity, assume that all choice variables take non-zero values).

C. Do r and w affect s_t and ℓ_t? If so how? { The easiest way to answer this is to solve for the optimal value for each variable.}

3. Suppose that the rate of depreciation depends on the intensity in which output is produced. Specifically, let δ be time varying, $\delta_t = D(y_t)$, where D is function that is increasing in output-per-effective labor unit (in mathematical notation, D'>0). The model determines aggregate values of output (Y) and capital (K) from a capital accumulation equation: $\frac{dK_t}{dt} = I_t - \delta K_t$, a constant returns-to-scale production function: $Y_t = F(K_t, A_t L_t)$ which has positive and diminishing marginal products, and growth of labor (L) and technology (A) given by $\frac{dL_t}{dt} = nL_t$ and $\frac{dA_t}{dt} = gA_t$ where g and n are positive constants. Assume this is a closed economy with no government in which savings is a fixed fraction of income. Let s be the savings rate.

A. Calculate the elasticity of steady state k with respect to s. (Write your answer in terms of the elasticity of y with respect to k, among other factors.) Show that the elasticity of steady state k with respect to s is greater than, less than, or equal to that elasticity from the Solow model?

B. Derive the linear dynamic equation used to analyze the speed of adjustment of k to its steady state. Show that the speed of adjustment for this model is faster, slower or the same as the Solow model? (Assume that the steady state the value of δ in this model is equal to the value of the exogenous depreciation rate from the Solow model.)

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2. A. Show in the standard (i.e. original) Solow model that the farther that k is below to its steady state value (in percentage terms) the larger is the growth rate of k.

B. Carefully define the concepts of convergence and conditional convergence. (In you answer, use an appropriate graph to illustrate what"convergence" means.) How well does the empirical evidence support or refute each of these concepts.

3. Instead of assuming the savings rate is a constant, let it be given by $s_t = s(y_t/y_*)$, where s is an increasing function of the ratio of y (output per effective labor unit) to its steady state value. In other words, $s' > 0$. Assume the elasticity of s_t with respect to (y_t/y_*) is less than one. Otherwise this is a fairly standard model: $Y_t = F(K_t, A_t L_t)$ with positive and diminishing marginal

products, $\frac{dL_t}{dt} = nL_t$, $\frac{dA_t}{dt} = gA_t$, $\frac{dK_t}{dt} = I_t - \delta K_t$, $I_t = S_t$, $\frac{S_t}{Y_t} = s(y_t / y_*)$ and in the steady state the elasticity of y with respect to k (capital per effective labor unit) is 1/3.

A. Calculate the elasticity of steady state k with respect to δ. (Write your answer in terms of the elasticity of f(k) with respect to k, among other factors.) Prove that this effect is always positive or negative or zero.

B. Derive the speed of adjustment of k to its steady state. Does this modification to the savings rate make the speed of convergence better or worse than the Solow model in terms of matching empirical evidence on the speed of adjustment? (Assume the steady state value of s in this model is equal to the value of the exogenous savings rate used in Solow's model.)

C. Prove that the speed of adjustment **ALWAYS** has the correct sign (that is, a sign which guarantees that the economy will converge to the steady state).

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1. Write an essay that answers the following question: Does the simple Solow growth model fit well with the data? In your answer, be sure to explain important predictions made by this theory and describe to what extent the evidence supports or refutes the basic model.

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2. Consider an overlapping generations model where consumers have the option to work a portion of their day (ℓ). Let utility be given by

$$\ln(c_{1t}) + b[\ln(1 - \ell_t)] + \frac{\ln(c_{2,t+1})}{1+\rho}$$

where consumers are subject to two budget constraints:

$$s_t + c_{1t} = w_t \ell_t \quad \text{and} \quad c_{2,t+1} = s_t(1 + r_{t+1})$$

where consumers choose consumption (c_{1t}), savings (s) and labor effort (ℓ) when young and consumption when old ($c_{2,t+1}$). Real wages and interest rates are taken as given by the consumer.

A. Set up the Lagrangian for the consumer to maximize utility subject to these resource constraints. (Often the two constraints are consolidated into a single constraint, but not necessarily.)

B. Derive first order conditions for this optimization problem.

C. How do interest rates and real wages affect c_{1t}, s_t, ℓ_t, and $c_{2,t+1}$. { Probably the easiest way to do this is to solve for optimal values as functions of these variables.}

3. In contrast to the original Solow growth model, suppose government expenditures (G) contribute to production because government spending makes private capital (K) and labor (L) more productive. Write the production function as:

$$Y_t = K_t^{\alpha} L_t^{\beta} G_t^{\gamma} A_t \quad \text{where} \quad \alpha + \beta + \gamma = 1.$$

Assume for simplicity that the government runs a balanced budget where taxes (T) are always equal to government expenditures. Consumers save (S) a fraction (φ) of disposable income:

$$S_t = \phi(Y_t - T_t).$$

Assume the standard capital accumulation equation holds where δ is the depreciation rate:

$$I_t = \frac{dK_t}{dt} + \delta K_t.$$

As usual this is a closed economy where savings equals investment. Let $L_t = L_0 e^{nt}$ and $A_t = A_0 e^{at}$ where a and n are constants. Assume all constants are positive.

A. Re-write each equation in per-units of effective labor, using lower case letters (y, k, g, i and s) to describe per-unit of effective labor. For example, $y_t = \frac{Y_t}{A_t L_t}$.

B. Derive the steady-state condition for k in terms of parameters and the level of government spending per-unit of effective labor.

C. How does g affect k in the steady state? In other words, calculate the derivative of k with respect to g in the steady state.

D. What is the steady-state (i.e. long-run) growth rate of Y?