Interpreting Permanent and Transitory Shocks to Output
When Aggregate Demand May Not Be Neutral in the Long-run

John W. Keating

University of Kansas
Department of Economics
213 Summerfield Hall
Lawrence, KS 66045
e-mail: jkeating@ku.edu
phone: (785)864-2837
fax: (785)864-5270

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Abstract: This paper studies a widely used statistical model of permanent and transitory shocks to output using a set of arguably more plausible structural assumptions. Many economic theories are inconsistent with the standard assumption that aggregate demand shocks have no long-run effect on the level of output. In contrast, this paper reinterprets that statistical model for output by assuming an aggregate supply shock that raises output will cause the price level to fall and an aggregate demand shock that initially raises output will cause the price level to rise. No assumption is made about the long-run effect of aggregate demand on output. Based on these assumptions, four primary results are obtained. These results are used to give structural interpretations to findings from the empirical literature. The paper concludes by discussing potentially important directions for future research.

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I. Introduction

Economists often investigate structural relationships using statistical models that decompose time series into permanent and transitory components. Much of this research has dealt with questions about aggregate real output. While initial studies employed univariate methods, more recent work almost always uses multivariate models. Blanchard and Quah (1989) developed one of the first multivariate models, and there have been numerous applications and extensions of their approach. In order for Blanchard and Quah’s decomposition to obtain permanent output shocks that identify the effects of aggregate supply and transitory output shocks that identify the effects of aggregate demand, the structure must satisfy a number of conditions including: (i) the long-run aggregate supply curve is vertical; (ii) aggregate demand shocks do not affect long-run supply; (iii) the dynamic structure is invertible; and (iv) shocks to supply and demand are uncorrelated.

Some researchers have been concerned about particular aspects of Blanchard and Quah’s bivariate approach. Lippi and Reichlin (1993) argued against invertibility and developed methods for handling non-invertible structures. Cover, Enders and Hueng (forthcoming) questioned the assumption that aggregate demand and supply shocks are uncorrelated, and they abandoned this restriction in favor of an alternative identifying assumption. Another concern is that Blanchard and Quah’s model only allows for a single aggregate supply shock and a single aggregate demand shock; if there are multiple types of either shock their bivariate specification could be misspecified. Blanchard and Quah (1989) show in an appendix the conditions that must hold for their approach to successfully identify the effects of aggregate demand and supply when an economy has multiple types of either shock. Faust and Leeper (1997) extend this idea in a number of important directions.

Another issue involves the use of the unemployment rate in this decomposition. Bordo (1993), Bayoumi and Eichengreen (1994), Karras (1994) and Keating and Nye (1998) replaced unemployment with price data. This substitution is based on the fact that textbook aggregate demand and supply theory is...
often used to justify the statistical model and this theory is nearly always formulated in terms of output and the price level. To explain the behavior of unemployment requires that a labor market structure be appended to aggregate demand and supply.

A second advantage of using price data in the statistical model is that a theory often predicts that different kinds of supply shocks will have qualitatively the same effect on each of the variables, along with a similar prediction for different aggregate demand shocks. Supply shocks typically cause output and price to move in opposite directions while demand shocks typically cause output and price to move in the same direction, at least for a significant portion of time after the shock occurs. On the other hand, the unemployment rate may respond in a fundamentally different way to different types of supply shocks. For example, an increase in labor supply raises output and the unemployment rate. An increase in labor demand also raises output but may initially lower the unemployment rate. If an economy experiences different supply shocks with fundamentally different effects on variables such as this, then results from Blanchard and Quah (1989) and Faust and Leeper (1997) show that the decomposition based on unemployment will not identify the effects of shocks to aggregate demand and supply. I will focus on models that use price data in place of unemployment, however, under certain conditions some of my results are invariant to this choice of variables.

This paper is primarily concerned with the possibility that aggregate demand shocks may not be neutral in the long run. While aggregate demand neutrality has often been considered a reasonable working hypothesis, it is inconsistent with many different macroeconomic theories. If any of these theories are relevant, the primary structural assumption used to justify Blanchard and Quah’s decomposition will be invalid. Can anything useful be learned by examining their statistical model under structural assumptions that allow for possible non-neutralities?

This paper is motivated by four findings from the empirical literature. First, many different studies have found that impulse responses based on postwar data are consistent with textbook theory.
Typically, permanent shocks behave like aggregate supply and temporary shocks behave like aggregate demand when the sample period begins after World War II. However, this result is not robust to all time periods. A second empirical finding is that a permanent increase in output is associated with an increase in the price level for most of the prewar economies studied by Keating and Nye (1998), where prewar defines the sample period ending right before World War I. These prewar responses to permanent shocks are inconsistent with the simple textbook model of aggregate supply and they are fundamentally different from the postwar results. A third finding, obtained from Keating and Nye (1998,1999) is that the output variance explained by permanent shocks tends to be larger in the pre-1914 period than in the postwar. The fourth finding is that the immediate effect on output of a permanent shock exceeds the long-run effect in most estimates with pre-World War I data. I define this impulse response characteristic as short-run overshooting. This type of response was found by Keating and Nye (1998,1999), Francis and Ramey (2003) and Bordo, Lane and Redish (2004) in samples that include 19th century data. Estimates from postwar subsamples did not find short-run overshooting in output responses.

I use a set of plausible structural assumptions to interpret the permanent and transitory shock decomposition. But instead of the standard assumption that aggregate demand shocks are neutral in the long run, I use inequality constraints on the dynamic responses of variables to structural shocks. Specifically I assume an aggregate supply shock that permanently raises/lowers output will cause the price level to fall/rise and an aggregate demand shock that initially raises/lowers output causes the price level to rise/fall. The long-run effect of aggregate demand on output is not constrained. Based on these assumptions, the aforementioned empirical findings can be given structural interpretations.

This paper shows that the tendency for price and output to move in the same direction in response to a permanent shock implies that aggregate demand shocks had positive long-run output effects for a number of prewar economies. This hypothesis is also supported by the short-run overshooting of output to permanent shocks in prewar economies and by the finding that permanent shocks typically explain a
greater share of output variance in the prewar period than in the postwar. Finally I show that the impulse responses in the statistical model are consistent with textbook theory over a range of positive and negative values for the parameter describing the long-run effect of aggregate demand on output. This result calls into question the interpretation of postwar findings as evidence that changes in aggregate demand are neutral in the long run. The paper concludes by discussing implications of these findings and potentially useful topics for future research.

II. The Structure and the Statistical Model

Under certain conditions a statistical model can be used as a means for discovering structural relationships. In this section I define the statistical model and the structure in terms of the dynamic responses of variables to shocks, or moving average representations as they are known in time series analysis. The structural moving average representation describes the dynamic response of each variable to each structural shock. For the statistical moving average representation, I use Blanchard and Quah’s decomposition of output into temporary and permanent shocks. This decomposition is obtained by imposing a particular set of identification restrictions on the reduced-form parameters of a VAR. This section introduces the VAR model and characterizes each of these moving average representations.

A. The VAR

In general, the VAR representation exists and is unique, and can be written as:

$$\beta(L) \Delta X_t = e_t$$  \hspace{1cm} (1)

where $X_t = (Y_t, P_t)'$ is the vector of variables consisting of the log of output and the log of the price level, $e_t$ is the vector of residuals, $\Delta$ is the first difference operator and $\beta(L) = I - \beta_1 L - \beta_2 L^2 \ldots - \beta_k L^k$ represents the coefficients in the VAR with the identity matrix and each $\beta_j$ for $j=1, 2, \ldots, k$ a $2 \times 2$ matrix.
and with $\kappa$ the number of lags in the VAR. Deterministic features such as constants, deterministic trends or time-based dummy variables that might be essential for conducting a valid empirical analysis have been omitted without loss of generality. A popular way of modeling economic data that are subject to permanent shocks is by first differencing the time series.

This specification is different from Blanchard and Quah because the second variable is the change in the logarithm of the price level, instead of the unemployment rate. This transformation of price is approximately equal to the rate of inflation. If the choice of second variable does not alter the identification restrictions for the statistical model or the theoretical assumptions used to interpret the model, then the output results in this paper are independent of that choice.

**B. The Structural Moving Average Representation**

Assume the economic structure has the following moving average representation:

$$\Delta X_t = \theta(L)\varepsilon_t$$  \hspace{1cm} (2)

where $\varepsilon_t = (\varepsilon_t^S, \varepsilon_t^D)'$ is a vector of shocks to aggregate supply and aggregate demand,

$$\theta(L) = \theta_0 + \theta_1 L + \theta_2 L^2 + ... = \sum_{j=0}^{\infty} \theta_j L^j$$

specifies the dynamic responses of $\Delta Y$ and $\Delta P$ to these structural shocks and each $\theta_j$ is a 2×2 matrix of structural parameters, with $\theta_j = \begin{bmatrix} \theta_{YS}^j & \theta_{YP}^j \\ \theta_{PS}^j & \theta_{PD}^j \end{bmatrix}$ for all $j$.

If supply and demand shocks are assumed uncorrelated, as is common in structural VAR modeling, then it is convenient to normalize these shocks to have variances equal to one: $E\varepsilon_t \varepsilon_t' = I$.

Recursive substitutions are used to transform equation (2), the system for first differenced variables, into a system in $X$, from which the dynamic responses of variables to structural shocks are obtained:

$$\frac{\partial X_t}{\partial \varepsilon_{t-k}} = \sum_{j=0}^{k} \theta_j \equiv \Phi_k.$$  \hspace{1cm} (3)
I define $\Phi_k$ as the k-th partial sum of $\theta(L)$ parameters, a definition that will be useful when making many of the points in this paper. Note that $\Phi_k$ is a 2×2 matrix:

$$
\Phi_k = \begin{bmatrix}
\Phi_{ks} & \Phi_{yd} \\
\Phi_{ps} & \Phi_{pd}
\end{bmatrix}
$$

(4)

where $\Phi_{vi} = \sum_{j=0}^{k} \theta^{vi}_j$ for $v=Y,P$ and $i=S,D$. While economists wish to estimate the responses in (3), this paper is concerned about structures that may cause the statistical model to inconsistently estimate the structural effects.

By letting $k$ go to infinity in (3) we obtain:

$$
\lim_{k\to\infty} \left( \frac{\partial X_1}{\partial \varepsilon_{t-k}} \right) = \sum_{j=0}^{\infty} \theta_j = \theta(1),
$$

(5)

where the last equality comes from setting $L=1$ in $\theta(L)$. The $\theta(1)$ matrix is written as:

$$
\theta(1) = \begin{bmatrix}
\Theta_{ys} & \Theta_{yd} \\
\Theta_{ps} & \Theta_{pd}
\end{bmatrix},
$$

(6)

with parameters that indicate the long-run effect of a supply or a demand shock on price or output.5

C. The Statistical Model’s Moving Average Representation

Let the moving average representation for the statistical model be written as:

$$
\Delta X_t = C(L)\mu_t
$$

(7)

where $\mu_t = (\mu_t^p, \mu_t^T)'$ is the vector of permanent and transitory shocks, respectively, and $C(L) = C_0 + C_1 L + C_2 L^2 + ... = \sum_{j=0}^{\infty} C_j L^j$ are the impulse responses of $\Delta X$ to these shocks with $C_j$ a 2×2 matrix for all non-negative integer values of $j$. In nearly all applications, the permanent and transitory shocks are assumed uncorrelated. Under that assumption, the variance of each shock in the
statistical model is conveniently normalized to one: \( E\mu_t\mu_t' = I \).

Applying the method of recursive substitution to equation (7) yields the impulse responses of output and price to the permanent and transitory shocks:

\[
\frac{\partial X_t}{\partial \mu_{t-k}} = \sum_{j=0}^{k} C_j .
\]

(8)

The long-run effects of these shocks on variables are obtained by letting \( k \) go to infinity in (8):

\[
\lim_{k \to \infty} \left( \frac{\partial X_t}{\partial \mu_{t-k}} \right) = \sum_{j=0}^{\infty} C_j = C(1) ,
\]

(9)

with \( C(1) \) representing the sum of coefficients in \( C(L) \). This matrix, written as:

\[
C(1) = \begin{bmatrix}
C_{YP} & 0 \\
C_{PP} & C_{PT}
\end{bmatrix}
\]

(10)

consists of parameters reflecting the long-run response of price or output to a permanent or a transitory shock. \( C(1) \) is made lower triangular by setting \( C_{YT} = 0 \), and as can be seen from (9), this restriction forces a temporary shock to have no long-run effect on output:

\[
\lim_{k \to \infty} \left( \frac{\partial Y_t}{\partial \mu_{t-k}} \right) = C_{YT} = 0 .
\]

III. Relationships between these Two Moving Average Representations

The statistical model’s moving average representation will not be identical to the structural moving average representation when the identification restrictions are not valid structural restrictions. However, the relationship between these two representations may still be informative. An easy way to derive this relationship is to map each of the moving average representations into the VAR. A VAR is a system of equations in which each variable is a function of lagged endogenous variables and a serially uncorrelated error. The statistical decomposition is transformed into the VAR by multiplying equation (7)
by $C_0 C(L)^{-1}$. The structure is transformed into the VAR by multiplying equation (2) by $\theta_0 \theta(L)^{-1}$, assuming the structure is invertible. These two mappings determine how VAR residuals:

$$e_t = C_0 \mu_t = \theta_0 e_t$$

(11)

and VAR coefficients:

$$\beta(L) = C_0 C(L)^{-1} = \theta_0 \theta(L)^{-1}$$

(12)

are functions of the statistical model and the structure. These equations will be used to characterize the relationship between coefficients from the statistical model and structural parameters, if $\Theta_{yd}=0$ or not.

Given equation (11) and the identity covariance matrix assumption for the shocks in each moving average representation, the covariance matrix for residuals:

$$\Sigma_e = C_0 C'_0 = \theta_0 \theta'_0,$$

(13)

is written as a function of short-run parameters from the structure and as a function of short-run coefficients from the statistical decomposition. A relationship between the statistical decomposition, the structure and $\beta(1)$, the sum of VAR coefficients matrix, is obtained by setting $L=1$ in equation (12):

$$\beta(1) = C_0 C(1)^{-1} = \theta_0 \theta(1)^{-1}.$$

(14)

The first identity in equation (14) yields: $C_0 = \beta(1) C(1)$. Inserting this expression into the first identity in equation (13) and simplifying yields:

$$C(1) C(1)' = \beta(1)^{-1} \Sigma_e \beta(1)^{-1}.$$  

(15)

This equation illustrates a well-known method for estimating parameters in this statistical model. Given that $C(1)$ is triangular, the $C(1)$ parameters can be obtained by the appropriate Cholesky decomposition of (15). And once these parameters are known, the dynamic response of each variable to each shock is obtained from the VAR.$^6$
Next the relationship between the statistical decomposition and the structure is investigated. Notice that the last identify in (14) yields: \( C_0 = \theta_0 \theta(1)^1 C(1) \). Insert this equation into the second equality of (13), and simplify to obtain:

\[
C(1)C(1)' = \theta(1)\theta(1)' .
\]  

(16)

The standard assumption from textbook theory is that aggregate demand shocks are long-run neutral, which is captured by \( \Theta_{YD} = 0 \) in the structure. This condition, along with the assumptions that the structure is invertible and that the structural shocks are orthogonal to one another, implies that the permanent and transitory shocks identify the dynamic effects of supply and demand, respectively. This implication is seen by noting that when \( \Theta_{YD} = 0 \), \( \theta(1) \) is lower triangular and therefore equation (16) yields \( C(1) = \theta(1) \) because the lower triangular factor of a symmetric matrix is unique. In this case, \( C(1) \) identifies the long-run effects of aggregate demand and aggregate supply on output and price. This last identity is combined with equation (12) which maps these two representations into the VAR coefficients to show \( C(L) = \theta(L) \). Naturally, the use of valid identifying restrictions permits the statistical model to obtain the complete dynamic response of each variable to each structural shock.

This paper is concerned with the possibility of \( \Theta_{YD} \neq 0 \). Using the second identities from equations (12) and (14), it is easy to derive the relationship between the statistical model’s impulse responses and the structural responses:

\[
C(L) = \theta(L)\theta(1)^{-1}C(1)
\]

(17)

or equivalently:

\[
C_j = \theta_j \theta(1)^{-1}C(1) \text{ for all } j.
\]

(18)

Using the definitions of \( \theta(1) \) and \( C(1) \) from (6) and (10), respectively, in equation (16), it is straightforward to determine how coefficients from the statistical model are related to long-run structural parameters.
\[ C_{YP} = \left( \Theta_{YS}^2 + \Theta_{YD}^2 \right)^{\frac{1}{2}}, \quad C_{PP} = \frac{\Theta_{YS} \Theta_{PS} + \Theta_{YD} \Theta_{PD}}{C_{YP}} \quad \text{and} \quad C_{PT} = \frac{\Theta_{YS} \Theta_{PD} - \Theta_{YD} \Theta_{PS}}{C_{YP}}. \]

From these three equations, the following calculation is made:

\[ \theta(1)^{-1} C(1) = \begin{bmatrix} \Theta_{YS} & -\Theta_{YD} \\ \Theta_{YD} & \Theta_{YS} \end{bmatrix} . \quad (19) \]

Insert (18) into (8), the equation describing impulse responses from the statistical model, and then use the definition of structural responses from (3) to show that:

\[ \frac{\partial X_t}{\partial \mu_{t-k}} = \sum_{j=0}^{k} C_j = \sum_{j=0}^{k} \theta_j \theta(1)^{-1} C(1) = \Phi_k \theta(1)^{-1} C(1) . \quad (20) \]

Substituting (19) and (4) into (20) yields:

\[ \frac{\partial X_t}{\partial \mu_{t-k}} = \begin{bmatrix} \Phi_k^{YS} & \Phi_k^{YD} \\ \Phi_k^{PS} & \Phi_k^{PD} \end{bmatrix} \begin{bmatrix} \Theta_{YS} & -\Theta_{YD} \\ \Theta_{YD} & \Theta_{YS} \end{bmatrix} . \quad (21) \]

Equation (21) shows the general relationship between the statistical model’s impulse response functions and the structural parameters. If \( \Theta_{YD} \neq 0 \), the statistical model’s coefficients are nonlinear functions of structural parameters instead of being consistent estimates of the structure. But, in spite of this inconsistency, the paper will show how this misspecified statistical model can still be used to infer important information about the economy, provided other valid structural assumptions are available.

**IV. Structural Assumptions**

If we are unable or unwilling to take a stand on some features of the structure, it is impossible, in general, to give structural interpretations to empirical models. More to the point, economists are unable
to infer how the impulse responses to permanent or transitory shocks are related to the structure without assumptions about how the economy operates. Blanchard and Quah, along with many others, have taken the position that aggregate demand is long-run neutral to interpret their statistical model. Even when this assumption is incorrect, Blanchard and Quah’s decomposition identifies a statistical model of permanent and transitory shocks to output. However, if aggregate demand is not assumed long-run neutral with respect to real output, alternative structural assumptions are required in order to give the statistical model an economic interpretation.

Fortunately other assumptions are available. Economic theory often places bounds on the qualitative responses of variables to structural shocks. For example, most theories predict that a beneficial aggregate supply shock will raise output and have a negative effect on the price level:

A1: \[
\frac{\partial Y_t}{\partial e_{t-k}^S} = \Phi_{k}^{YS} > 0 \quad \text{for all } k;
\]

A2: \[
\frac{\partial P_t}{\partial e_{t-k}^S} = \Phi_{k}^{PS} < 0 \quad \text{for all } k.
\]

These assumptions are weak enough to allow for the possibility that supply shocks also shift the aggregate demand curve. If supply shocks cause both curves to shift in the same direction, then assumption A2 requires that the demand curve not shift by as much as supply. Our assumptions permit supply shocks to have wealth effects, however, A2 places an upper bound on the magnitude of those effects. An example of an aggregate supply factor that shifts both curves in the same direction is a permanent increase in productivity. This beneficial supply shock raises capital’s expected future marginal product which increases investment demand, causing aggregate demand to shift to the right.

There is some debate in economic theory about the long-run effect of aggregate demand on output. Since the long-run aggregate supply curve is vertical in all modern macroeconomic theories, that means a shift in the aggregate demand curve will not have a long-run effect on output unless it has a
permanent effect on some aggregate supply factor. There are a number of different theories that predict aggregate demand may be non-neutral in the long run. Certain theories predict that an increase in aggregate demand will cause output to be higher in the long run while other theories find the opposite effect. Examples of theories in which aggregate demand may not be long-run neutral are:

- **Non-Superneutrality:** A permanent increase in the growth rate of money raises or lowers output in the long run depending on particular features of the structure;
- **Long-Run Fiscal Policy Effects:** An increase in government spending crowds-out or crowds-in investment in the long-run, affecting the stock of capital and long-run aggregate supply; Changes in marginal tax rates have supply-side effects;
- **Hysteresis:** The natural unemployment rate depends on past levels of the unemployment rate, allowing aggregate demand to affect the natural rate. Consequently, aggregate demand influences the level of output in the long run;
- **Coordination Failures:** Coordination problems often yield multiple equilibria which can permit aggregate demand to affect the long-run equilibrium position of the economy;
- **Destabilizing Price Flexibility:** Rather than bring about general equilibrium, deflation may make the economy worse off, and a faster decline in prices can exacerbate the adverse effects.

Irrespective of the ultimate consequence for output, I assume that a positive aggregate demand shock raises output for some time after the shock occurs:

\[
A3: \quad \frac{\partial Y_t}{\partial e_{D,t-k}} = \Phi_k^{YD} > 0 \quad \text{for } k=0,1,...,K \quad \text{with } 0 < K < \infty. 
\]

This assumption allows for output to possibly decline at some point in time after a beneficial aggregate demand shock. One way for this to occur is if aggregate demand has a negative long-run effect on output. Another way is if output cycles around its steady state for some time before settling down.
I also assume that an aggregate demand shock which initially causes output to rise has a positive effect on the price level:  

\[ \Phi_{k}^{PD} > 0 \quad \text{for all } k. \]

If a beneficial aggregate demand shock also shifts aggregate supply to the left ($\Theta_{YD} < 0$), then A4 will hold because the movement of each curve raises the price level. On the other hand, if a beneficial shock to aggregate demand also shifts aggregate supply to the right ($\Theta_{YD} > 0$), then for A4 to hold the supply curve must not shift by as much as demand does.

These four assumptions may not hold for every theory, but they are consistent with most economic structures. Importantly, they do not rule out long-run neutrality of aggregate demand because $\Theta_{YD}$ is unconstrained. And as we will see, some of our results hold under even weaker conditions.

V. Results

Using permanent and transitory shock decompositions with pre-World War I data for countries that have relatively long time series, Keating and Nye (1998) found that a permanent increase in output is associated with an increase in the price level in 8 of the 10 countries. In 5 cases, this positive price response is statistically significant. This evidence strongly rejects the textbook structure underlying Blanchard and Quah’s (1989) decomposition because if permanent shocks are supply shocks they should move price and output in opposite directions. Is a rejection of the structural hypothesis all that can be inferred or does this evidence tell us something interesting about the structure of prewar economies?

**Proposition 1:** Given assumptions A1, A2 and A4, if a permanent increase in output is associated with an increase in the price level, then aggregate demand must have a positive effect on output in the long run.
Proof of Proposition 1:

From equation (21), the response of price to a permanent increase in output is

$$\frac{\partial P_i}{\partial \mu_{i-k}} = \frac{\Phi_{i-k}^{PS} \Theta_{YS} + \Phi_{i-k}^{PD} \Theta_{YD}}{C_{YP}}.$$

The condition on $\Theta_{YD}$ that makes this response positive is:

$$\Theta_{YD} > -\frac{\Phi_{i-k}^{PS} \Theta_{YS}}{\Phi_{i-k}^{PD}}$$

and the structural assumptions guarantee that the right side is positive. Q.E.D.

Keating and Nye (1998) speculated that the unusual prewar impulse responses might be evidence of this structural hypothesis. Proposition 1 shows that their conjecture is valid under certain structural conditions. Thus, in addition to rejecting the simple textbook model, most of their prewar price responses to a permanent increase in output also imply, under arguably more plausible structural assumptions, that aggregate demand had positive long-run output effects in prewar economies.

From the last equation in the proof it is clear that Proposition 1 only requires output to rise in the long run after a beneficial supply shock. Thus the proposition will hold under the considerably weaker assumption that output may fall, rise or remain unchanged for some time after a technological improvement occurs, before rising in the long run. This modified assumption is consistent with Basu, Fernald, and Kimball (2004), for example, who argue that output may initially decline following technological improvement. Consequently, the conclusion that aggregate demand shocks had positive long-run output effects in many prewar economies will hold under very general assumptions about the dynamic response of output to technology shocks.

Another empirical finding is that impulse responses for some economies are fundamentally different across the two time periods. For example, Keating and Nye (1998) found in prewar data that the
immediate effect on output of a permanent shock is larger than the long-run effect for 7 of the 10 countries in their study.13 This sort of response, defined as short-run overshooting, is not observed in postwar estimates from any of these countries. Using different empirical specifications, Francis and Ramey (2004) and Bordo, Lane and Redish (2004) have also observed short-run overshooting responses for output in samples that include prewar data. I will argue that the most plausible structural explanation for this response pattern is that output became permanently higher when aggregate demand increased.

The response of output to a permanent shock is taken directly from equation (21):

$$\frac{\partial Y_t}{\partial \mu_{t-k}} = \frac{\Phi_k^{YS} \Theta_{YS} + \Phi_k^{YP} \Theta_{YD}}{C_{YP}}.$$  

(22)

If \( \Theta_{YD} = 0 \) then the response of output to a permanent shock identifies the response of output to supply. I am not familiar with any economic theory in which supply shocks have a short-run effect on output that exceeds the long-run effect. Theory generally shows that output gradually rises in response to a permanent beneficial supply shock.14 Therefore, \( \Theta_{YD} = 0 \) is unable to explain short-run overshooting.

Now consider the case of \( \Theta_{YD} < 0 \). Along with A1, A3 and (22), this assumption implies:

$$\frac{\partial Y_t}{\partial \mu_{t-k}} < \Phi_k^{YS} \text{ for small } k,$$

because when \( \Theta_{YD} \) is negative the coefficient on \( \Phi_k^{YS} \) in (22) is less than one and the second term in the numerator from (22) is negative. Furthermore, for any non-zero \( \Theta_{VD} \):16

$$\lim_{k \to \infty} \left( \frac{\partial Y_t}{\partial \mu_{t-k}} \right) = C_{YP} = (\Theta_{YS}^2 + \Theta_{YD}^2)^{1/2} > \Theta_{YS}.$$

From the last two equations it is clear that when \( \Theta_{YD} \) is negative, a permanent output shock will have smaller short-run and larger long-run effects on output than an aggregate supply shock. Thus, if supply shocks don’t cause short-run overshooting in output, a permanent shock to output can not exhibit short-
run overshooting when aggregate demand has a long-run negative effect on output.

This leaves $\Theta_{yd}>0$ as the last case to consider. Equation (22) shows that, in general, the response of output to a permanent shock is a linear combination of the output responses to aggregate supply and aggregate demand. The coefficient on $\Phi_{k}^{YD}$ in equation (22) is positive if and only if $\Theta_{yd}$ is positive. Macroeconomic theories often predict an aggregate demand shock will have its peak effect on output after roughly a year or so, an effect that is qualitatively similar to the short-run overshooting observed in models with annual prewar data. Therefore, if an increase in aggregate demand has a long-run positive output effect that is sufficiently large, the response of output to a permanent shock could inherit short-run over-shooting behavior from the dynamic response of output to aggregate demand shocks.

Most research with permanent and transitory decompositions has used postwar data. The empirical results from that sample period are typically consistent with the aggregate supply interpretation of permanent shocks and the aggregate demand interpretation of transitory shocks. Therefore, economists often conclude that the textbook aggregate demand and supply model characterizes postwar economies.

Another interesting finding is that the amount of variance explained by permanent shocks to output tends to be larger in the pre-1914 period than in the post-World War II period. This finding is obtained by Keating and Nye (1999) who follow Blanchard and Quah and use the unemployment rate and by Keating and Nye (1998) who use inflation in place of the unemployment rate. This difference can only occur at finite horizons because as the forecast horizon goes to infinity permanent shocks will, by construction, explain 100% of the variance of output. Could variation in the long-run output effect of aggregate demand explain these differences in variance decomposition? The following proposition is used to address this question.

**Proposition 2.** If the aggregate demand and supply structure applies to two economies, demand shocks to Economy A are long-run neutral, demand shocks to Economy B may have a long-run effect on
output, and this is the only difference between these two economies, then the fraction of finite horizon output variance associated with permanent shocks is larger for Economy B if and only if aggregate demand has a long-run positive effect on output in Economy B.

Proof of Proposition 2: See Appendix B.

Suppose Economy A is a postwar country for which empirical results are usually consistent with textbook theory and Economy B be that same country in the prewar period. Assuming long-run neutrality holds in postwar economies, Proposition 2 shows that a positive long-run effect of aggregate demand on output in the prewar period could by itself explain the observed differences in output variance decomposition. Hence, the variance results provide additional support for the hypothesis that some economies in the late 19th and early 20th centuries experienced a permanent increase in output from aggregate demand shocks.17

Proving Proposition 2 is equivalent to showing that more output variance is explained by permanent shocks than by aggregate supply shocks, if and only if positive (negative) aggregate demand shocks permanently increase (decrease) output. The appendix discusses how certain negative values of $\Theta_{yd}$ could, in principle, also explain this variance relationship. However, these negative values can be ruled out by assuming a permanent shock always has a positive effect on output. Empirical results in Keating and Nye (1998,1999) provide strong evidence to support this assumption.

A criticism of the permanent and transitory shock literature is that few studies have formally tested the structural assumptions. Using these assumptions along with postwar data, researchers have typically obtained empirical evidence consistent with textbook theory. Of course, failure to reject a theory does not mean that the theory is necessarily correct. In most cases, the qualitative features of impulse response functions are used to determine consistency with a theory. An interesting question is: How are
the dynamic features of the statistical model influenced by the values of long-run structural parameters?

**Proposition 3:** Impulse responses to permanent and transitory shocks are consistent with the qualitative effects of textbook aggregate supply and aggregate demand shocks, respectively, over a range of positive and negative values for $\Theta_{YD}$.

Proof: Calculate bounds on $\Theta_{YD}$ such that the statistical model finds the shocks that permanently increase output cause price to fall and the temporary shocks that initially increase output cause price to rise. The following discussion presents and interprets these bounds.

Based on equation (21), it is easy to show that the temporary shock causes output and the price level to rise, respectively, when

$$\frac{\Phi_k^{YD} \Theta_{YS}}{\Phi_k^{YS}} > \Theta_{YD} > \frac{\Phi_k^{PD} \Theta_{YS}}{\Phi_k^{PS}}.$$  \hspace{1cm} (23)

Given assumptions A1 through A4, if $\Theta_{YD}$ is greater than the negative value on the right for all $k$ and less than the positive number on the left for small values of $k$ (i.e. non-negative $k$ that are less than $K$ from assumption A3), then the statistical model will yield impulse responses to temporary shocks that appear like the aggregate demand shocks from simple textbook macro models. Thus if our structural assumptions are valid and we observe output and price responses to temporary shocks that move in opposite directions for small $k$, that implies $\Theta_{YD}$ falls outside the range of values in (23) and therefore can not equal zero. In fact, Keating and Nye (1998) found that temporary shocks cause price and output to move in opposite directions for half of the prewar sample of countries. This finding can be interpreted as further evidence that aggregate demand is not long-run neutral for some prewar economies. Unfortunately that evidence alone does not indicate whether the long-run effect is positive or negative.  \hspace{1cm} 18
Next examine responses to a permanent shock. From equation (21), the permanent output shock will have a negative effect on price when:

$$-\frac{\Phi_{PS}^{k}\Theta_{YS}}{\Phi_{PD}^{k}} > \Theta_{YD}.$$ 

Given A1, A2 and A4, this expression sets a positive upper bound on $\Theta_{YD}$ under which the permanent shock to output will cause a drop in the price level. Proposition 1 addresses the eight pre-World War I economies which apparently violated this inequality.

Now determine the conditions under which the permanent shock will always have a positive effect on output. First note that this response is positive at long horizons, by construction, for any value of $\Theta_{YD}$. Next consider the case when $\Phi_{k}^{YD}$ is positive. Under this assumption, (21) is used to show that the response of output to a permanent shock is positive for finite $k$ when

$$\Theta_{YD} > \frac{-\Phi_{k}^{YS}\Theta_{YS}}{\Phi_{k}^{YD}}.$$ 

Thus $\Theta_{YD}$ must be greater than a negative number, given A1, A3 and the maintained assumption that $\Phi_{k}^{YD}$ is positive.

On the other hand, if $\Phi_{k}^{YD}$ is negative for certain values of $k$, the permanent shock will have a positive effect on $Y$ when:

$$\Theta_{YD} < \frac{-\Phi_{k}^{YS}\Theta_{YS}}{\Phi_{k}^{YD}}.$$ 

In this case, the inequality sign is reversed because of division by a negative $\Phi_{k}^{YD}$ and the right-hand side of the inequality is now positive. This new condition sets a positive upper bound on $\Theta_{YD}$. Thus a range of positive and negative values for $\Theta_{YD}$ that permits the permanent shock to have a positive effect on output for all $k$ has been established.

Based on the many empirical studies that have assumed long-run neutrality of aggregate demand,
this structural assumption appears to have a significant amount of credibility. Hence, it is natural to interpret empirical findings consistent with the textbook model as support for that model. But responses to temporary and permanent shocks will appear to be consistent with the effects of aggregate demand and supply shocks, respectively, for a range of values for $\Theta_{VD}$. While $\Theta_{VD}=0$ is included in that range of parameter values, Proposition 3 illustrates why the qualitative features of impulse response functions do not serve as a reliable basis for judging that the long-run neutrality assumption is valid. And if $\Theta_{VD}$ is not very close to zero, the impulse responses and variance decompositions from the statistical decomposition of output will almost certainly differ by a substantial margin from the effects of structural shocks.

VI. Discussion and Conclusions

This paper provides structural interpretations of four empirical findings obtained from decompositions of output into permanent and transitory shocks. An important conclusion is that beneficial/adverse aggregate demand shocks had permanent positive/negative effects on output in some prewar economies. A question for future research is: What caused aggregate demand to be non-neutral in these economies? While a number of potential structural explanations exist, some don’t seem very plausible based on the evidence.

For example, long-run real effects from permanent changes in the growth rate of money are not evident in the prewar sample period. For non-superneutrality to potentially be a factor, persistent or permanent changes in the growth rate of the money supply would need to occur. Such changes would likely cause standard tests to find unit roots in inflation, money growth and the growth rates of other nominal quantities, even if some other type of model provided a better description of the stochastic process for these nominal changes. In fact, unit roots are easily rejected in prewar inflation rates for the 10 countries studied in Keating and Nye (1998). In contrast, various studies have found unit roots in postwar inflation. Thus, while there is no prewar evidence against long-run superneutrality, non-
superneutrality may be feature of some postwar economies.20

Crowding-out or crowding-in from government spending and supply-side effects from changes in tax rates are alternative ways for aggregate demand to have permanent output effects. But these mechanisms are also more likely to be relevant for postwar economies. Tax rates and government shares of output were both very small in the prewar period, and large-scale government involvement in the macroeconomy did not occur prior to the Great Depression.

Hysteresis in the labor market provides another mechanism by which aggregate demand may have long-run output effects.21 If a recession causes permanent loss in the stock of human capital, then the marginal product of labor permanently declines, causing a reduction in labor demand which could raise the natural unemployment rate. In that case, an adverse shock to aggregate demand causes a permanent reduction in full-employment output and a positive aggregate demand shock has the opposite effect. Persistent changes in aggregate demand could then make the unemployment rate appear to have a unit root.22 In fact, tests on postwar unemployment rate data frequently fail to reject a unit root, while the unit root hypothesis is easily rejected for unemployment rates from the prewar sample period for the countries studied by Keating and Nye (1999).23 This unit root evidence appears to be inconsistent with the hysteresis explanation of prewar results.

On the other hand, coordination failure theories provide a plausible explanation of permanent output effects from aggregate demand in the prewar period. A coordination failure may occur when economic decisions have strategic complementarities or spillover effects.24 Examples of these effects are when financial market liquidity depends on the number of market participants or when the utility of a communications device depends on how many others use that device. Coordination failure economies often exhibit multiple equilibria. Consequently, a positive aggregate demand shock may push the economy to a higher level of economic activity, while a negative demand shock could permanently reduce output. An empirical observation consistent with this hypotheses is that transactions costs were falling.
throughout the nineteenth and twentieth centuries. Transactions costs for businesses and individuals were lowered by developments in financial intermediation, advances in transportation and improvements in communications technology. This reduction in transactions costs over time may have transformed 19th Century coordination failure economies into modern structures that appear to be well-described by textbook macroeconomic theory.

Destabilizing price flexibility is another plausible explanation. Macroeconomic theory usually predicts that more rapid price adjustment causes aggregate demand to have smaller output effects. However, there are certain mechanisms by which falling prices will push the economy farther away from full employment. In this case, faster price adjustment could make the economy experience a persistent decline in output following a negative shock to aggregate demand. Fisher (1933) explained how deflation and debt may combine to produce adverse aggregate outcomes. Keynes (1936), Tobin (1975) and DeLong and Summers (1986) have emphasized how falling prices might raise real interest rates and reduce spending, an effect that may be particularly significant when nominal interest rates are close to their lower bound. Consistent with this hypothesis is the evidence, discussed in Calomiris and Hubbard (1989) for example, that prices were more flexible in the pre-1914 period than in the postwar and also the fact that deflationary episodes were quite common during this earlier period.

This discussion points to the need for empirical work that directly tests these hypotheses or any other hypothesis that might be offered to explain why aggregate demand was not long-run neutral in prewar economies. Some of the evidence appears consistent with destabilizing price flexibility and some with coordination failure, but more definitive studies are clearly needed. One reason for determining the cause for non-neutral aggregate demand is to gain a better understanding of pre-World War I economies. A second reason is that the same structural mechanisms may still be relevant in modern economies. While the postwar results are largely consistent with textbook theory, Proposition 3 illustrates why this evidence does not necessarily mean the prewar mechanisms have become irrelevant in the postwar. The finding
that price levels fall with a permanent increase in output in nearly all postwar estimates suggests that if the prewar effects continue to be relevant, their influence seems to have waned. However, if certain non-neutral mechanisms were found still relevant, that information would enable a better understanding of modern economies.

It is also possible the mechanisms that affected pre-1914 economies are no longer important and that fundamentally different structural mechanisms may invalidate long-run neutrality of aggregate demand in the postwar. This concern is plausible given that hysteresis, non-superneu-trality of money and permanent output effects from fiscal policy are more likely to be relevant in postwar economies. An important implication of Proposition 3 is that testing for long-run neutrality is advisable. While some testing methods have been developed, new procedures or improvements to currently available methods would be beneficial.28 A failure to reject neutrality in postwar economies would elicit greater confidence in the structural interpretations drawn from the vast empirical literature that has employed neutrality assumptions. However, rejections of long-run neutrality would call for new empirical studies on many important macroeconomic questions.

The methods in this paper can be extended in a number of promising directions. The bivariate permanent and transitory shock decomposition of any variable is amenable to these methods.29 Structural assumptions are, of course, contingent on the variable that is decomposed into these two types of shocks and on the other variable used in the model. The methods in this paper can also be modified and extended to models with more than two shocks.30 This modification allows a researcher to investigate the structural implications of empirical models that identify multiple kinds of permanent and multiple kinds of temporary shocks to output or any other variable.31

This paper provides a deeper understanding of permanent and transitory shock decompositions of output. It is well-known that inconsistent estimates can be obtained when key identification assumptions are invalid. Nevertheless, when other identifying assumptions are available, we can still use these
decompositions to infer important facts about the structure of an economy.
Notes


2. An important advantage of a multivariate approach is that it utilizes more information in the decomposition. Other advantages are that multiple structural relationships can be estimated and that the permanent and transitory shocks can be orthogonal to one another. See Quah (1992) for further results.


5. While $\Phi_4 = \theta(1)$, in the following analysis it is useful to distinguish between finite horizon effects, $\Phi_k$ for finite $k$, and long-run effects, $\theta(1)$.

6. The complete dynamic response of each variable to each shock is obtained analytically by inserting the estimate of $C(1)$ into the first equality from (14), solving for $C_o$, inserting $C_o$ into the first equality in (12), and solving for $C(L)$. In practice, dynamic responses are generated by simulating the model.


8. These solutions implicitly take the positive roots of $C_{yp}$ and $C_{pt}$. For $C_{pt}$ to be positive, $\Theta_{ys} \Theta_{pd} > \Theta_{yd} \Theta_{ps}$ is required. Given the structural assumptions found in the next section of the paper, this inequality holds for any positive value of $\Theta_{yd}$, and probably most, if not all, plausible negative values of this parameter. But even if $\Theta_{yd}$ is so negative that the previous inequality no longer holds, responses to permanent shocks would be unaffected and so the key results in the paper would not change.

9. This point is a corollary of correlation not always implying causation.

10. Faust (1998), Uhlig (2001) and Canova and De Nicoló (2002) use sign restrictions to help identify a model’s parameters. Waggoner and Zha (2003) discuss inference problems that can arise from inappropriately normalizing equations in a simultaneous system, giving some attention to the implications of their analysis for the work done by Faust and Uhlig. Waggoner and Zha point out that recursive models are immune to such problems. Since I use sign restrictions to interpret a recursive statistical model, my analysis is also immune to these normalization concerns.
11. We could have made the weaker assumption that this price response is non-negative. Having $\Phi_{k}^{PD} = 0$ for small $k$ would allow us to interpret the statistical model under the assumption that prices are sticky in the short run following an aggregate demand shock. This possibility only yields one new insight: A permanent increase in output will unambiguously lower the price level for small $k$. There is some evidence that prices are sticky in the postwar and in nearly all postwar estimates the price level falls with a permanent increase in output. Furthermore, in 8 of 10 prewar countries price rises with a permanent increase in output. Thus most of the prewar empirical findings are inconsistent with
These empirical findings support the view of some economists that price adjustment was relatively fast in the prewar and relatively slow in the postwar. See the discussion in Calomiris and Hubbard (1989) and their references to differences in price adjustment between these two periods.

12. Section 4 in Keating and Nye (1998) argues that problems with the quality and consistency of pre-1914 data are unable to explain this unusual finding.

13. Short-run overshooting is found in prewar samples for the US, UK, Sweden, Japan, Italy, Germany and France, but not for Canada, Denmark and Norway. Appendix C provides the impulse responses for all prewar and postwar estimates from Keating and Nye (1998). This appendix was removed from the published version of that paper. I include these figures here because they show that short-run overshooting occurred for most of the prewar economies and none in the postwar.

14. Cyclical dynamics may also occur as the economy approaches its steady state, but these dynamics occur with a delay in dynamic economic models and so do not resemble short-run overshooting.

15. All of the results concerning short-run overshooting hold even when a beneficial supply shock causes output to initially decline.

16. This inequality also proves that the coefficient on $\Phi_{k}^{YS}$ in (22) is bounded between zero and one for any non-zero $\Theta_{YD}$.

17. While this finding supports the idea that aggregate demand had a long-run positive effect on output in prewar economies, it does not rule out the possibility that additional factors may have increased the postwar output variance associated with temporary shocks. For example, some economists believe that stabilization policy is actually destabilizing to an economy. In this case, more activist postwar monetary policies would raise the output variance explained by aggregate demand shocks, and therefore the temporary shocks from the statistical model would, ceteris paribus, explain more output variance. While this hypothesis can explain the variance decomposition results, it does not explain why permanent output shocks would raise price in the long-run in most of the prewar estimates.

18. Keating and Nye (1998) obtain this result for developing prewar economies and not for developed postwar economies. Morling (2002) obtains a similar result for many developing postwar economies. This evidence suggests that the stage of development may play a role in this pattern of impulse responses.

19. These unit root test results were not reported in the published version of that paper.


22. Technically, the unemployment rate is bounded from above and below and therefore can not have a unit root. But permanent shifts in the natural rate could lead standard testing procedures to accept a unit root.
23. Test results for the pre-1914 sample period are available on request.


25. See Wallis and North (1986), for example. I thank John Nye for pointing out this paper to me, although he may not necessarily agree with my interpretations.


27. The postwar sample of Keating and Nye (1998) ended in 1994. Since then, Japan has experienced periods of persistent deflation. More recent Japanese data might be useful for testing the destabilizing price flexibility hypothesis in a modern economy.

28. Fisher and Seater (1993) and King and Watson (1997) developed methods for testing long-run neutrality propositions, but neither approach is easily extended to models with more than two endogenous variables.

29. Output-per-hour by Gali (1999), stock prices by Cochrane (1994) and inflation by authors cited in endnote 20 are only a few examples.

30. Keating (2005) presents a multivariate extension of the basic approach developed here. That paper also provides a structural investigation of empirical work that decomposes inflation into permanent and transitory shocks, under the assumption that permanent movements in inflation may not reflect purely exogenous changes in the growth rate of the money supply when monetary policy is endogenous.

References


Cover, James Peery, Walter Enders and C. James Hueng “Using the Aggregate Demand-Aggregate Supply Model to Identify Structural Demand-Side and Supply-Side Shocks: Results Using a Bivariate VAR,” forthcoming in the *Journal of Money, Credit and Banking*.


Appendix A: Recursive Substitution

Equation (2) can be written as:

\[ X_t = X_{t-1} + \theta(L)e_t. \]

This relationship holds for any time period, and so:

\[ X_{t-1} = X_{t-2} + \theta(L)e_{t-1}. \]

Substituting this expression for \( X_{t-1} \) into the first equation yields:

\[ X_t = X_{t-2} + \theta(L)e_t + \theta(L)e_{t-1}. \]

Then a similar substitution is made for \( X_{t-2} \), followed by \( X_{t-3} \), etc., to obtain:

\[ X_t = X_0 + \sum_{k=0}^{t-1} \theta(L)e_{t-k}. \]

This equation can also be written as:

\[ X_t = X_0 + (\theta_0 e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots) + (\theta_0 e_{t-1} + \theta_1 e_{t-2} + \theta_2 e_{t-3} + \ldots) + \ldots \]

Matching up the coefficients on each \( \epsilon_{ij} \) yields:

\[ X_t = X_0 + \theta_0 e_t + (\theta_0 + \theta_1) e_{t-1} + (\theta_0 + \theta_1 + \theta_2) e_{t-2} + \ldots, \]

from which equation (3) obtains.

The same basic method is used with the statistical model to obtain equation (8).
Appendix B: Proof of Proposition 2.

Structural assumptions:
I. For Economy A, the textbook aggregate demand and supply model describes the structure, hence the permanent and transitory shock decomposition identifies structural effects.
II. For Economy B, aggregate demand shocks have permanent effects on output, hence the permanent-transitory shock decomposition is unable to identify effects of aggregate demand and supply.
III. Both economies have identical short-run and intermediate-run structures.

Calculate the k-step forecast error for each economy:

For Economy A, the decomposition of permanent and transitory shocks identifies the structure, and so equation (2) is used to derive the k-step forecast error:

\[ X_t - E_{t-k} X_t = \sum_{j=0}^{k-1} \Phi_j \varepsilon_{t-j} = \sum_{j=0}^{k-1} \begin{bmatrix} \Phi^YS_j & \Phi^YD_j \\ \Phi^PS_j & \Phi^PD_j \end{bmatrix} \begin{bmatrix} \epsilon^S_{t-j} \\ \epsilon^D_{t-j} \end{bmatrix}. \]  

For economy B, the permanent and transitory shock decomposition fails to identify the structure because aggregate demand has a permanent effect on output. Equations (17) and (7) are combined and used to determine the k-step forecast error for this case:

\[ X_t - E_{t-k} X_t = \sum_{j=0}^{k-1} \Phi_j \theta(1)(C(1)^{-1}) \mu_t = \frac{\sum_{j=0}^{k-1} \begin{bmatrix} \Phi^YS_j & \Phi^YD_j \\ \Phi^PS_j & \Phi^PD_j \end{bmatrix} \begin{bmatrix} \Theta^YS_j & -\Theta^YD_j \\ -\Theta^YD_j & \Theta^YS_j \end{bmatrix} \begin{bmatrix} P^T_{t-j} \\ \mu^T_{t-j} \end{bmatrix}}{\left(\Theta^2^YS + \Theta^2^YD\right)^{1/2}}. \]  

Since these two economies are different in the long run, the \( \Phi_j \) parameters can’t be the same for all possible \( j \). Given Assumption III, however, the \( \Phi_j \) parameters are the same for \( j=0,1,2,\ldots,k-1 \) where \( k \) is some finite integer.

The fraction of k-step forecast error variance for output associated with the permanent shocks for Economy A is obtained from equation (i):

\[ \frac{\sum_{j=0}^{k-1} (\Phi^YS_j)^2}{\sum_{j=0}^{k-1} (\Phi^YS_j)^2 + (\Phi^YD_j)^2}, \]  

and for Economy B it is obtained from equation (ii):

\[ \frac{\sum_{j=0}^{k-1} \left(\Phi^YS_j \Theta^YS_j\right)^2 + \left(\Phi^YD_j \Theta^YD_j\right)^2 + 2\Phi^YS_j \Theta^YS_j \Phi^YD_j \Theta^YD_j}{\sum_{j=0}^{k-1} \left(\Phi^YS_j^2 + \Phi^YD_j^2\right)\left(\Theta^YS_j^2 + \Theta^YD_j^2\right)}. \]
The question is: What conditions on $\Theta_{YD}$ guarantee that permanent shocks explain a larger fraction of output variance in Economy B? Multiplying by positive denominators, collecting terms and simplifying, one can show that equation (iv) is greater than equation (iii) when:

$$2\Theta_{YS}\Theta_{YD}\sum_{j=0}^{k-1}\Phi_{j}^{YS}\Phi_{j}^{YD} + \Theta_{YD}^2 \sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2 > 0$$

It is convenient to divide by $\Theta_{YS}^2$, and factor the expression as follows:

$$\left(\frac{\Theta_{YD}}{\Theta_{YS}}\right)^2 \left(2\sum_{j=0}^{k-1}\Phi_{j}^{YS}\Phi_{j}^{YD} + \sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2\right) > 0. \quad (v)$$

From assumptions A1 and A3 we know that $\sum_{j=0}^{k-1}\Phi_{j}^{YS}\Phi_{j}^{YD}$ is positive. However, $\sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2$ may be zero, negative or positive depending on the relative importance of supply and demand shocks for output during the first $k$ periods. Consider each of these cases:

1. If $\sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2 = 0$, $\frac{\Theta_{YD}}{\Theta_{YS}} > 0$ satisfies (v);

2. If $\sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2 < 0$, $0 < \frac{\Theta_{YD}}{\Theta_{YS}} < \frac{-2\sum_{j=0}^{k-1}\Phi_{j}^{YS}\Phi_{j}^{YD}}{\sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2}$ satisfies (v);

3. If $\sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2 > 0$, (v) is satisfied by either:

$$\frac{\Theta_{YD}}{\Theta_{YS}} > 0;$$

OR $$\frac{\Theta_{YD}}{\Theta_{YS}} < \frac{-2\sum_{j=0}^{k-1}\Phi_{j}^{YS}\Phi_{j}^{YD}}{\sum_{j=0}^{k-1}\left(\Phi_{j}^{YD}\right)^2 - \left(\Phi_{j}^{YS}\right)^2} < 0.$$

In 3 of these 4 conditions $\frac{\Theta_{YD}}{\Theta_{YS}}$ is positive, and therefore $\Theta_{YD}$ is positive. However, the last condition is satisfied by a negative parameter value. If we can show that these negative $\Theta_{YD}$ values are irrelevant, that will complete a proof that $\Theta_{YD}$ is positive.
I will show that one way to rule out these negative values is to assume that permanent output shocks always have a positive effect on output. This is true for all model estimates in Keating and Nye (1998), with the exception of Japan using the 1972 to 1994 sample period. In this exceptional case, the largest root in the VAR is complex and explosive and so the output response will end up negative at some points with probability one. This unusual response suggests some type of specification error is involved and so we discount that finding (but even in this case, the response of output to a permanent shock is positive for the first 7 years, so the following result holds for this sample when $k-1 \leq 7$).

Assuming the output response to a permanent shock is always positive places a lower bound on $\Theta_{YD}$, a negative number that we will show is not as negative as the values of $\Theta_{YD}$ in the last condition for case 3. Hence, the negative values of $\Theta_{YD}$ that satisfy (v) are so negative that a permanent shock would cause output to fall at some point in the impulse response, a condition that is ruled out by the evidence.

The response of output to a permanent shock is given by equation (22), and if the responses for the first $k$ periods are positive this equation implies:

$$\frac{\Theta_{YD}}{\Theta_{YS}} > -\frac{\Phi_{YS}^{j}}{\Phi_{YD}^{j}} \quad \text{for } j=0,1,\ldots,k-1.$$

for some finite $k$. Let $\rho_{j} = \frac{\Phi_{YS}^{j}}{\Phi_{YD}^{j}}$. Each $\rho_{j}$ is positive because of assumptions A1 and A3. Define $\rho_{*}$ as the minimum over all $\rho_{j}$ for $j = 0, 1, 2,\ldots, k-1$. Based on the previous inequality we can see that if $\frac{\Theta_{YD}}{\Theta_{YS}}$ were smaller than $-\rho_{*}$, some portion of output’s response to a permanent shock would be negative. Since output does not fall in response to a permanent shock, $-\rho_{*}$ sets a lower bound for $\frac{\Theta_{YD}}{\Theta_{YS}}$. The negative values in case 3 are ruled out if:

$$-2 \sum_{j=0}^{k-1} \Phi_{YS}^{j} \Phi_{YD}^{j} < -\rho_{*}, \quad \text{(vi)}$$

To show that (vi) holds, use the definition of $\rho_{j}$ to eliminate $\Phi_{YS}^{j}$ and, given $\sum_{j=0}^{k-1} \left[ \left( \Phi_{YD}^{j} \right)^{2} - \left( \Phi_{YS}^{j} \right)^{2} \right]$ is positive in this case, manipulate (vi) into the following inequality:

$$\sum_{j=0}^{k-1} \left( 2\rho_{j} - \rho_{*} + \rho_{*} \rho_{j}^{2} \right) \left( \Phi_{YD}^{j} \right)^{2} > 0.$$

Since $\rho_{j}$ is positive for all $j$ and $\rho_{j} \geq \rho_{*}$ for all $j$, this inequality is unquestionably true. Therefore (vi) holds, ruling out all cases of $\Theta_{YD} < 0$ that satisfy (v), and completing the proof of Proposition 2.
Appendix C: Impulse Responses from Keating and Nye (1998)

These figures were not published in the final version of Keating and Nye (1998).

This appendix provides impulse responses of the price level and real output to temporary and permanent shocks to output for the 10 countries studied by Keating and Nye (1998).

The point estimate is a solid line and the 90 percent confidence interval is enclosed within a pair of dashed lines.

Responses from prewar and postwar samples are included.

When a structural break test rejected parameter stability in VAR coefficients for the full sample estimate, we also estimated VARs for subsamples that have stable parameters.
Real Output Responses to Permanent Output Shocks
Price Level Responses to Permanent Output Shocks